

# Doppler controlled dynamics of a mirror attached to a spring

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A laser beam directed at a mirror attached onto a flexible mount extracts thermal energy from its mechanical Brownian motion by Doppler effect. For a normal mirror the efficiency of this Doppler cooling is very weak and masked by laser shot-noise. We find that it can become really efficient using a Bragg mirror at the long wavelength edge of its band stop. The opposite effect opens new routes for optical pumping of mechanical systems: a laser pointing at a Bragg mirror and tuned at its short wavelength edge induces amplification of the vibrational excitation of the mirror leading eventually to its self-oscillation. This new effects rely on the strong dependency of the Bragg mirror reflectivity on the wavelength.

Radiation pressure and even electromagnetic fluctuation of the vacuum are known to affect the dynamics of mirrors attached to a spring [1, 2] but less is known about relativistic effects of light on such macroscopic mechanical oscillators. Recent results show that opto-mechanical couplings can be exploited in a deformable Fabry-Perot arrangement to cool down or enhance the mechanical fluctuations of spring loaded tiny mirrors [3]. This effect exploits the fact that inside a Fabry-Perot cavity the photon-pressure reacts to any change in the mirror position with a time delay given by the cavity photon storage time [3, 4, 5, 6, 7]. These cavity effects, and notably the cavity laser cooling, have much analogy with Doppler cooling of atoms yet they do not require relativistic effects for their interpretation. In this work we describe a mechanism for true Doppler modified dynamics of a mirror thermally and mechanically anchored to a spring. Without the use of any cavity, this mechanism could allow to set optically the mirror into a regime of mechanical self-oscillation, simply by pointing a laser at it.

A laser beam at wavelength and power  $P$  illuminates the mirror of reflectivity  $R(\lambda)$  as schematized in fig. 1. The radiation pressure acting on a moving mirror varies in proportion to the Doppler shift. We show here that because of this dynamics, the Brownian fluctuation of the suspended mirror loses its energy to the electromagnetic field. We found that the efficiency of Doppler cooling can be much increased using a mirror with a large and negative gradient of  $R(\lambda)$ . We also show that an amplification of the vibrational excitation can be reached for positive large enough gradient of  $R(\lambda)$ . Large gradient of reflectivity are usually found at the edges of the band-stop of a Bragg mirror.

The movable mirror of mass  $m$  is subject to a force  $F_{ph}$  due to radiation pressure. Its position  $x$  and velocity  $v$  of the center of mass of the mirror obey the Newton's equation of motion, namely  $m(dv/dt) + m\Gamma v + Kx = F_{ph} + F_{th}$ . The Langevin force  $F_{th}$  is introduced here in order to account for the Brownian fluctuation of the center mass of the mirror coupled to a thermal bath at temper-

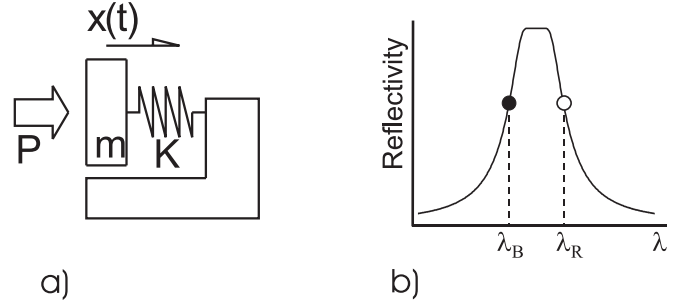


FIG. 1: (a) Schematics of a mirror of reflectivity  $R(\lambda)$  and mass  $m$  mechanically attached and thermally anchored to a spring of rigidity  $K$ . The friction mechanisms internal to the spring are responsible for losses of mechanical energy at a rate  $\Gamma$ . A laser of power  $P$  is directed at the mirror from the left. (b) Schematics of the reflectivity wavelength dependency of a Bragg mirror. The gradients of  $R$  are maximized at the edges  $\lambda_B$  and  $\lambda_R$  of the mirror band stop.

ature  $T$ . The damping rate  $\Gamma$  of the mirror mechanical fluctuation is a factor characteristic of the spring holding the mirror. The momentum transferred to the mirror per photon is given by  $\hbar(k_0 - k_R)$ . Here the incoming photons have their wave vector  $k_0 = 2\pi/\lambda$  and the reflected ones have  $k_R$  oriented in the opposite direction. In the reference frame of the laboratory, when the mirror moves away from the light source, the reflected photons have their momentum reduced by Doppler effect such  $k_R = -k_0(1 - 2v/c)$ . The radiation pressure is given by the rate of photon momentum transfer to the mirror and is also reduced by Doppler effect such that  $F_{ph} = R dN/dt \hbar(k_0 - k_R)$  where  $dN/dt$  is the number of impinging photon per unit time. The dependency of the radiation pressure on  $v$  is  $F_{ph} = 2R(dN/dt)\hbar k_0(1 - v/c)$  and by making use of the laser power  $P = \hbar k_0 c(dN/dt)$  it is also  $F_{ph} = (2RP/c)(1 - v/c)$ . In addition, when the mirror reflectivity is a function of  $\lambda$  as it is the case for a Bragg mirror operated near its band-stop edge,  $R$  also depends on the mirror velocity through the Doppler effect. In this case, we expand  $R[\lambda(v/c)]$  in the experimentally relevant limit of  $v/c \ll$

1. To the first order in  $k$ , the reflectivity is  $R(v/c) \simeq R_0 + (v/c)\lambda(dR/d\lambda)$ , so in the same way, to the first order in  $v/c$  the radiation pressure is  $F_{ph} \simeq (2R_0P/c)[1 - v/c + (v/c)(\lambda/R_0)(dR/d\lambda)]$ . Using this expression in the equation of motion and grouping the velocity terms together we obtain an effective Newton equation of motion  $m(dv/dt) + m\Gamma_{eff}v + Kx = F_{ph,0} + F_{th}$  with a constant radiation pressure  $F_{ph,0} = 2R_0P/c$  and a Doppler modified damping rate

$$\Gamma_{eff}/\Gamma = 1 + (2R_0P/mc^2\Gamma)[1 - (\lambda/R_0)(dR/d\lambda)] \quad (1)$$

The constant force  $F_{ph,0}$  only shifts the average position of the center of mass and will be ignored in solving the effective equation of motion. For  $dR/d\lambda \leq 0$ , the optical contribution to the effective dissipation term  $\Gamma_{eff}$  takes energy away from the mechanical Brownian fluctuation and turns it irreversibly into electromagnetic energy through Doppler effect. This amounts to cooling of the Brownian fluctuations of the mirror. We now determine the temperature of the vibrational motion of the mirror. For a harmonic oscillator the equipartition theorem links the temperature to the time averaged amplitude  $\langle xx^* \rangle$  of the Brownian fluctuation, namely  $(1/2)k_B T_{eff} = (1/2)K\langle xx^* \rangle$ . In experimental conditions it is not the temporal dependency  $x(t)$  but rather its spectral distribution  $x_\omega$  which is typically measured. The spectrum  $x_\omega$  is in fact a Fourier transform of  $x(t)$ . A mathematically convenient property of Fourier transformation is that the time averaged value  $\langle x(t)x^*(t) \rangle$  term equals the frequency averaged value  $\langle x_\omega x_\omega^* \rangle$ . In Fourier space the Newton's effective equation is  $(-m\omega^2 + im\Gamma_{eff}\omega + K)x_\omega = F_{th,\omega}$ , from which we obtain the spectrum  $x_\omega = (F_{th,\omega}/m)/(-\omega^2 + i\Gamma_{eff}\omega + K/m)$ . Here for a non-absorbing mirror, the spectral decomposition  $F_{th,\omega}$  of the thermal fluctuation driving force does not depend on the light and is evaluated from the situation in dark. A reasonable guess about the nature of the driving force  $F_{th,\omega}$  is that there is no preferred frequency for the thermal fluctuations in the range of the mirror mechanical vibrational frequencies. This is reasonable at vibrational mechanical frequencies much lower compared to typical phonon frequencies with high density of phonon modes within the mirror material (THz). Within this approximation we assume the spectral power density  $S_{th}$  in thermal excitation of the mirror to be frequency independent, such that for any given frequency window  $d$  the amplitude of the thermal driving force obeys  $F_{th,\omega}F_{th,\omega}^* = S_{th}d\omega$ . Using the expression of  $x_\omega$  given above we obtain after some algebra  $(1/2)K\langle x_\omega x_\omega^* \rangle = \pi S_{th}/(4m\Gamma_{eff})$ . The left hand side this equation is  $(1/2)k_B T_{eff}$  as prescribed by the equipartition theorem and we end up with  $(1/2)k_B T_{eff} = \pi S_{th}/(4m\Gamma_{eff})$ . Since the spectral power density  $S_{th}$  is not light dependent, in dark we also have  $1/2k_B T = \pi S_{th}/(4m\Gamma)$ . Comparing both expressions we obtain  $T/T_{eff} = \Gamma_{eff}/\Gamma$ , showing that an increase in  $\Gamma_{eff}$  leads

to cooling. This conclusion is premature because so far we ignored the effect of photon shot noise. For a given laser intensity we need to include the fundamental shot-noise power fluctuation that induces a corresponding shot-noise in the radiation pressure and hence driving an additional vibrational fluctuation or an added vibrational temperature. This added fluctuation could counteract the Doppler cooling. This fluctuation force  $F_{shot}$  is very much analogous to  $F_{th}$  but is proportional to square root of the laser power. Here  $F_{shot,\omega}F_{shot,\omega}^* = S_{shot}d\omega$  with a frequency independent power density  $S_{shot} = (2R_0/c)^2(Ph\nu)/2\pi$  where  $h\nu$  is the photon energy. Assuming no correlation between the Brownian and shot noise, we obtain the new prescription  $(1/2)KT_{eff} = \pi(S_{th} + S_{shot})/(4m\Gamma_{eff})$  leading to the expression for the effective vibrational temperature:

$$T_{eff} = (T + pT_{phot})/(1 + p(1 - \nabla R)) \quad (2)$$

where we defined i) the unitless reduced laser power  $p = 2R_0P/(mc^2\Gamma)$ , ii) the reflectivity gradient  $\nabla R = (\lambda/R_0)(dR/d\lambda)$  and iii) an effective photon temperature  $k_B T_{phot} = R_0 h\nu/2$ . The power dependent term in the denominator originates from the Doppler effect while the one in the numerator is due to the photon shot-noise. We see that for a mirror with a constant reflectivity  $dR/d\lambda = 0$ , the Doppler effect tends to lower the effective temperature while the shot noise terms increases it. For such a mirror, the condition  $T_{eff} < T$ , namely for cooling is only possible when  $T_{phot} < T$  or  $R_0 h\nu/2 < k_B T$ . For visible or near-infrared photons and for high reflectivity mirrors  $R_0 \sim 1$ , this condition cannot be satisfied at room temperature. For a mirror with a large and negative reflectivity gradient however, such that  $\nabla R < 0$ , Doppler cooling of the vibrational mode becomes possible when  $T_{phot}/(1 - \nabla R) < T$  or  $R_0 h\nu/[2(1 - \nabla R)] < k_B T$ . This gives a stringent condition for the reflectivity gradient. At room temperature using 1 eV photons on a mirror with  $R_0 \sim 0.5$ , namely for  $T_{phot} = 2900$  K, the condition on the reflectivity gradient would be  $-\nabla R > 8.7$ . Experimentally this can be obtained using a Bragg reflector and a photon wavelength tuned at the higher wavelength-edge of the band-stop of the reflectivity (fig. 1b). Values as high and negative as  $(\lambda/R_0)(dR/d\lambda) = -10^5$  are in fact within experimental reach in the visible and near infrared [9]. Doppler cooling saturates using large enough laser power at  $T_{min} = T_{phot}/(1 - \nabla R)$ . With the numerical example above this would be 29 mK.

Another interesting aspect of Bragg mirrors is that they offer also the opportunity to set the gradient  $\nabla R$  positive enough to reach  $\Gamma_{eff} \leq 0$  in equation (1). In this regime the mirror gains energy and starts to self-oscillate under the illumination of a CW laser. It enters possibly in a regime of nonlinear dynamics similar to the one predicted by F. Marquardt and coworkers [7] for deformable Fabry-Perot cavities. This effect would be interesting to

demonstrate in this context of cavity-less system. The gain condition implies using a laser power large enough so that  $P/\Gamma > mc^2/[2R_0(\nabla R-1)]$ . The energy term  $P/\Gamma$  represents the laser power averaged over a mechanical relaxation time constant  $1/\Gamma$ . This energy is compared to  $mc^2$ , the relativistic energy of the mirror so we anticipate already the Doppler induced optomechanic might be very weak for macroscopic mirrors. We recently prepared a gold mirror ( $dR/d\lambda \sim 0$ ) mounted on silicon nano-lever [8] with a mass in the  $10^{-15}$  Kg range. For such a mirror and for  $R_0 \sim 0.5$ , the condition for self-oscillation would be reached when  $P/\Gamma > 90/(\nabla R-1)$ . Using a damping rate  $\Gamma \sim 10 \text{ sec}^{-1}$  and  $\nabla R \sim 10^5$  this would imply using laser power of 90 mW in order to enter a regime of self-oscillation. In order to increase the Doppler effect we see from eq. (2) that one needs to decrease the mass  $m$ . For a Bragg mirror, this can only be done within bounds because photons probe the material periodicity within a finite penetration depth. The mass cannot arbitrarily be reduced by thinning the material, at some point the reflectivity and its gradient will degrade. Also the lateral dimensions of the reflector cannot be reduced much less than the diffraction limit [8]. In the visible range we anticipate that the smallest masses would be in the  $10^{-15}$  Kg range.

Now we compare the strength of Doppler-cooling with cavity cooling established in ref [3, 4, 5, 6]. A Fabry-Perot cavity of length  $L$  separating the mirrors and finesse  $F > 1$  stores electromagnetic energy with a typical ring-down time-constant  $\tau \sim (F/\pi)\tau_0$  where  $\tau_0 = L/c$  is the photon time of flight across the cavity.  $\tau$  is the typical time the cavity needs to build or loose energy upon a sudden change in laser power or in mirror separation. The photon pressure acting on the mirrors is proportional to the stored power so that the force acting on the mirror near a cavity resonance is not only enhanced by the cavity but also retarded with respect to the mirror separation fluctuation. Retarded terms amounts to velocity dependent force terms similarly to the Doppler effect discussed above. For a cavity with at least one of the two mirrors mounted on a spring, this retarded effect induces an optical modification of the mechanical damping rate and consequently a modification of the vibrational temperature the same way developed above. In order to cool the vibrational fluctuations of the mirror, it is necessary to detune slightly the laser wavelength to the red with respect to the cavity resonance [3, 4, 5, 6]. Optimum detuning  $\delta\{L/\lambda\}$  is obtained on the maximum slope of the dependency of electromagnetic energy stored in the cavity with respect to wavelength or mirror separation. This is the case when  $\delta\{L/\lambda\} = -1/(g\sqrt{3})$ . The reverse effect, namely opto-mechanical excitation, is obtained for blue detuning. We establish the extremal effective temperatures

$$T_{eff} \simeq (T + pgT_{phot})/[1 \pm p(L/\lambda)g^3/(g^2\omega_0^2\tau_0^2 + 1)] \quad (3)$$

Where  $g = 2F/\pi$  is proportional to the cavity finesse  $F$ . The sign depends on the side of the detuning with respect to a cavity resonance. For a given vibrational frequency  $\omega_0 = (K/m)^{1/2}$  it turns out that optimal cooling (or pumping) is obtained at the condition  $\omega_0\tau \sim 1$ .

In expressing equation (3) we made the reduced power  $p = 2R_0P/mc^2\Gamma$  appear explicitly in order to make a direct comparison with Doppler cooling. The term in the denominator is usually much larger than unity and we see that cooling efficiency using cavity effects can be made stronger by a factor as large as  $g^3(L/\lambda)$  than Doppler cooling. Already for a finesse as low as  $g = 10$  and for a length  $L = 20\,000\lambda$ , the cavity cooling can be made  $10^7$  more efficient than direct Doppler cooling and this considering equal mirror masses, laser powers and damping rates. The use of large finesse allows a significant amplification of laser cooling but at the same time imposes that the moving mirror be part of a Fabry-Perot cavity and this is not always convenient. We stress however the fact that all models derived so far describing cavity cooling do not include relativistic effects. While mimicking it, cavity cooling cannot be interpreted as Doppler cooling. For completeness we introduced Doppler effect in our formalism of cavity cooling and found that it gives rise to corrections to the cooling efficiency that are small for cavity finesse  $g \gg 1$  [10].

We finish this letter on an estimation of the cooling power involved with Doppler effect.

In dark, the mechanical fluctuation dissipates its thermal energy  $k_B T/2$  per mechanical degree of freedom and this at a rate  $\Gamma$ . The dissipated power is therefore  $(k_B T/2)\Gamma$  and is in equilibrium with the power that feeds the fluctuation. When the mirror reflects the laser light, the effective vibrational mode end-temperature is  $T_{eff}$ . When the vibrational mode is cooled down to a temperature  $T_{eff}$ , the steady state heat-load in the mirror is  $(k_B T_{eff}/2)\Gamma$ . Consequently in order to maintain a temperature  $T_{eff}$ , the optical cooling extracts energy from the fluctuations of the mirror position at a rate  $P_{cooling} = k_B(T - T_{eff})\Gamma/2$  which is always smaller than  $k_B T\Gamma/2$ . So the maximum cooling power is  $k_B T\Gamma/2$  both for Doppler and cavity cooling. This is in the range of  $10^{-18}$  Watts at room temperature for  $\Gamma$  in the  $10^3 \text{ sec}^{-1}$  range. This might appear as a very weak cooling power but it can be efficient enough to cool the lowest energy vibrational modes of an elastically suspended mirror since such modes are generally weakly coupled to the thermal bath.

In conclusion, we presented a simple formalism for laser Doppler cooling of the center mass fluctuation of a mirror attached to a spring. This effect is very weak but can become sizeable when the mirror reflectivity is made to depend strongly on the photon wavelength. We also

showed that effective temperature obtained through cavity cooling, in a formalism that does not include Doppler effect, mimics direct Doppler cooling but with a cavity amplification factor which is proportional to the third power of the cavity finesse and can easily reach ten orders of magnitudes. The reciprocal effect of Doppler cooling, namely Doppler optical pumping of the mirror motion, was also predicted. Interestingly enough, we showed that the use of an appropriate Bragg mirror should allow overcoming the shot noise limit and use directly the Doppler effect to set the mirror motion into self-oscillation. This could provide a very simple and non-invasive method to optically pump the motion of tiny mechanical resonators.

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